Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji

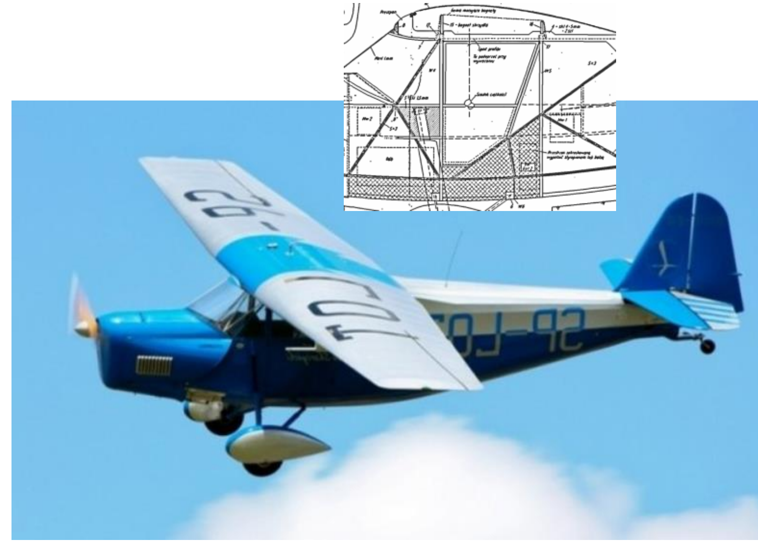
Metoda elementów skończonych (MES1)

Wykład 7C. 2D element pręta kratownicy

04.2024

Przykłady kratownic

Kratownica mostu

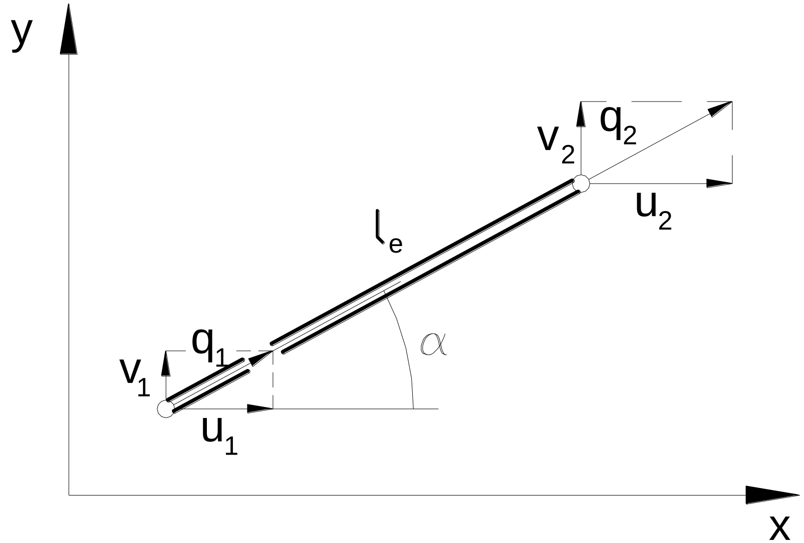


Kratownica kadłuba

Kratownica dźwigu

Kratownica dachu

Element skończony pręta kratownicy 2D



Lokalny wektor parametrów węzłowych: $\lfloor q \rfloor_e = \lfloor q_1, q_2 \rfloor_e$

Globalny wektor parametrów węzłowych:

$$\lfloor q_g \rfloor_e = \lfloor u_1, \upsilon_1, u_2, \upsilon_2 \rfloor_e$$

Transformacja wektora parametrów węzłowych:

$$q_i = u_i \cos\alpha + \upsilon_1 \sin\alpha \qquad (i = 1,2)$$

$$\{q\}_e = [T_k]\{q_q\}_e$$

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_e = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix} \begin{Bmatrix} u_1 \\ \upsilon_1 \\ u_2 \\ \upsilon_2 \end{Bmatrix}_e$$

Lokalna macierz sztywności pręta: $[k]_e = \dfrac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Energia sprężysta elementu:

$$U_e = \frac{1}{2}\underset{1\times 2}{\lfloor q \rfloor}\underset{2\times 2}{[k]_e}\underset{2\times 1}{\{q\}_e} = \frac{1}{2}\underset{1\times 4}{\lfloor q_q \rfloor_e^T}\underset{4\times 2}{[T_k]^T}\underset{2\times 2}{[k]_e}\underset{2\times 4}{[T_k]}\underset{4\times 1}{\{q_q\}_e}$$

$$U_e = \frac{1}{2}\lfloor q_q \rfloor_e [k_g]_e \{q_q\}_e$$

Globalna macierz sztywności pręta kratownicy:

$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

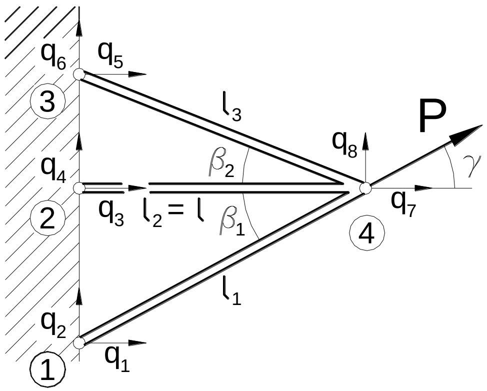
$s = \sin\alpha,\ c = \cos\alpha$

Przykład kratownica 3 prętowa

Element 1 nodes 1 and 4 slope angle $\alpha_1 = \beta_1$ length $l_1 = \dfrac{l}{\cos\alpha_1}$.

Element 2 nodes 2 and 4 slope angle $\alpha_2 = 0$ length $l_2 = \dfrac{l}{\cos\alpha_2}$.

Element 3 nodes 3 and 4 slope angle $\alpha_3 = -\beta_2$ length $l_3 = \dfrac{l}{\cos\alpha_3}$.



$$\left[k_{ij}\right]_e^1, \left[k_{ij}\right]_e^2, \left[k_{ij}\right]_e^3$$

$$\begin{bmatrix}
k_{11}^1 & k_{12}^1 & 0 & 0 & 0 & 0 & k_{13}^1 & k_{14}^1 \\
k_{21}^1 & k_{22}^1 & 0 & 0 & 0 & 0 & k_{23}^1 & k_{24}^1 \\
0 & 0 & k_{11}^2 & k_{12}^2 & 0 & 0 & k_{13}^2 & k_{14}^2 \\
0 & 0 & k_{21}^2 & k_{22}^2 & 0 & 0 & k_{23}^2 & k_{24}^2 \\
0 & 0 & 0 & 0 & k_{11}^3 & k_{12}^3 & k_{13}^3 & k_{14}^3 \\
0 & 0 & 0 & 0 & k_{21}^3 & k_{22}^3 & k_{23}^3 & k_{24}^3 \\
k_{31}^1 & k_{32}^1 & k_{31}^2 & k_{32}^2 & k_{31}^3 & k_{32}^3 & k_{33}^1 + k_{33}^2 + k_{33}^3 & k_{34}^1 + k_{34}^2 + k_{34}^3 \\
k_{41}^1 & k_{42}^1 & k_{41}^2 & k_{42}^2 & k_{41}^3 & k_{42}^3 & k_{43}^1 + k_{43}^2 + k_{43}^3 & k_{44}^1 + k_{44}^2 + k_{44}^3
\end{bmatrix}
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ q_7 \\ q_8 \end{Bmatrix}
=
\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ P\cos\gamma \\ P\sin\gamma \end{Bmatrix}$$

Warunki brzegowe:

$q_j = 0$ $j = 1,6$

$$EA\begin{bmatrix} \sum_{i=1}^{3} \dfrac{c_i^2}{l_i} & \sum_{i=1}^{3} \dfrac{s_i c_i}{l_i} \\ \sum_{i=1}^{3} \dfrac{s_i c_i}{l_i} & \sum_{i=1}^{3} \dfrac{s_i^2}{l_i} \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P\sin\gamma \\ P\cos\gamma \end{Bmatrix}$$

Dla: $\beta_1 = \beta_2 = \beta$ $\gamma = 0$

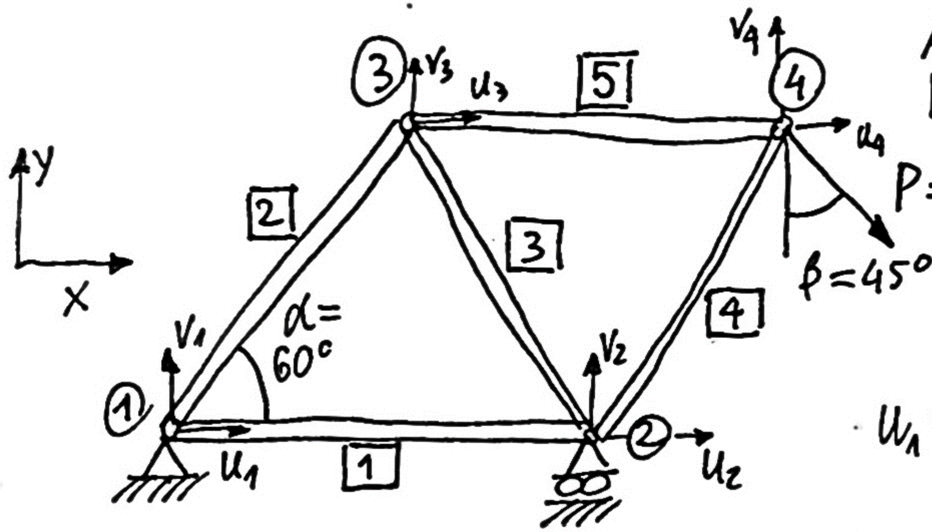
$$\frac{EA}{l}\begin{bmatrix} 1+2c^3 & 0 \\ 0 & 2s^2c \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

Dla: $c = \cos\beta$, $s = \sin\beta$

$$q_7 = \frac{Pl}{EA(1+2c^3)}$$

$$q_8 = 0$$

Przykład Zbuduj model MES 2 wymiarowej kratownicy. Znajdź przemieszczenia węzłowe, naprężenia, siły wewnętrzne i reakcje



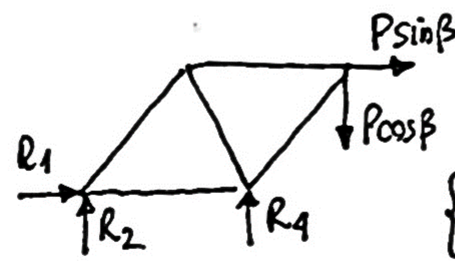
$E = 2\cdot 10^5$ MPa

$A = 100$ mm^2

$L = 1000$ mm

$u_1 = 0,\ v_1 = 0,\ v_2 = 0$

$$\underset{8\times 1}{\{q\}} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

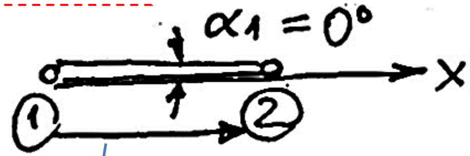


$$\underset{8\times 1}{\{F\}} = \begin{Bmatrix} R_1 \\ R_2 \\ 0 \\ R_4 \\ 0 \\ 0 \\ P\sin\beta \\ -P\cos\beta \end{Bmatrix}$$

$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

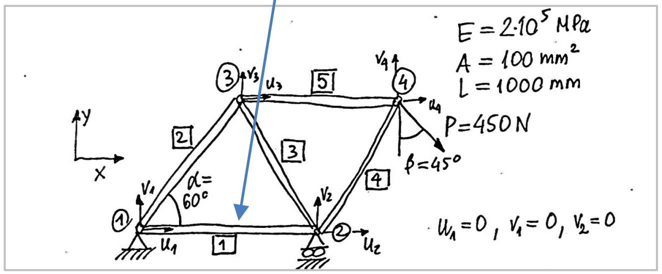
[1]



$c_1 = 1\ ,\ s_1 = 0\ ,\ \lfloor q_g \rfloor_1 = \lfloor u_1, v_1, u_2, v_2 \rfloor$ (1×4)

$$\underset{4\times4}{[k_g]_1} = \frac{EA}{l}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

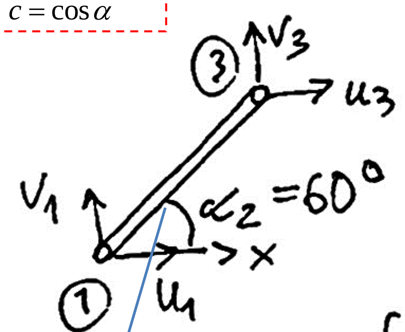
$$\underset{8\times8}{[k_g]_1^*} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

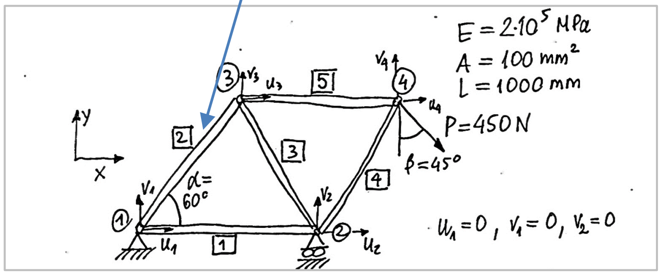
[2]



$$c_2 = \frac{1}{2},\quad s_2 = \frac{\sqrt{3}}{2},\quad \underset{1\times 4}{\lfloor q_g \rfloor_2} = \lfloor u_1, v_1, u_3, v_3 \rfloor$$

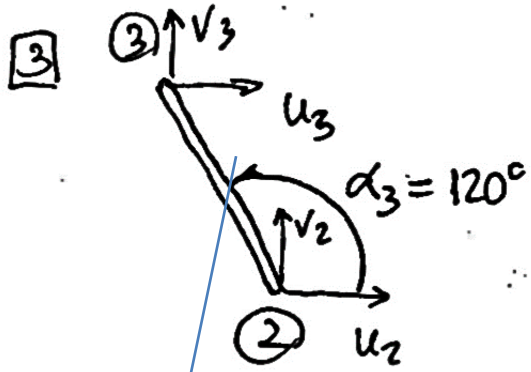
$$\underset{4\times 4}{[k_g]_2} = \frac{EA}{4l}\begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times 8}{[k_g]_2^*} = \frac{EA}{4l}\begin{bmatrix} 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

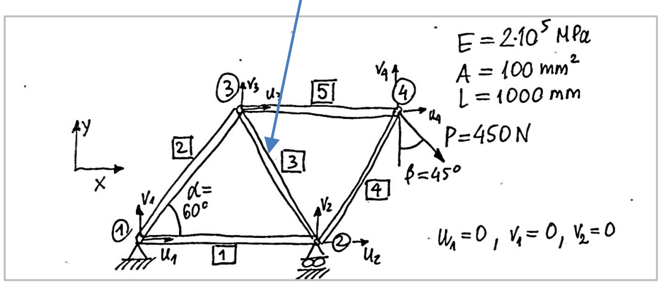
$s = \sin\alpha,\ c = \cos\alpha$



$C_3 = -\frac{1}{2}$, $S_3 = \frac{\sqrt{3}}{2}$, $[q_g]_3 = [u_2, V_2, u_3, V_3]$

$$\underset{4\times4}{[k_g]_3} = \frac{EA}{4l}\begin{bmatrix} 1 & -\sqrt{3} & -1 & \sqrt{3} \\ -\sqrt{3} & 3 & \sqrt{3} & -3 \\ -1 & \sqrt{3} & 1 & -\sqrt{3} \\ \sqrt{3} & -3 & -\sqrt{3} & 3 \end{bmatrix}$$

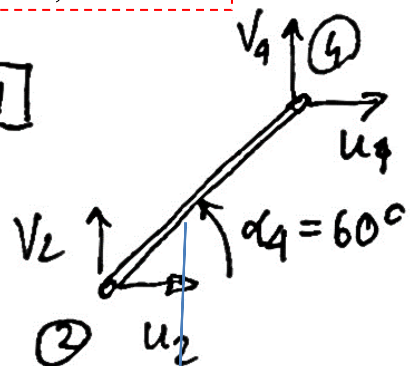
$$\underset{8\times8}{[k_g]_3^*} = \frac{EA}{4l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\sqrt{3} & -1 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & -1 & \sqrt{3} & 1 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{3} & -3 & -\sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

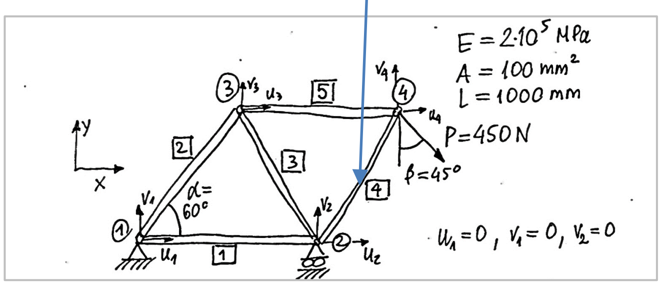
[4]



$$c_4 = \frac{1}{2},\quad s_4 = \frac{\sqrt{3}}{2},\quad \lfloor q_g \rfloor_4 = \lfloor u_2, v_2, u_4, v_4 \rfloor$$

$$\underset{4\times4}{[k_g]_4} = \frac{EA}{4l}\begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_4^*} = \frac{EA}{4l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

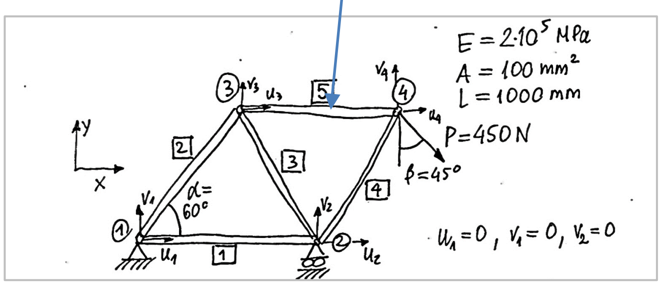
$s = \sin\alpha,\ c = \cos\alpha$

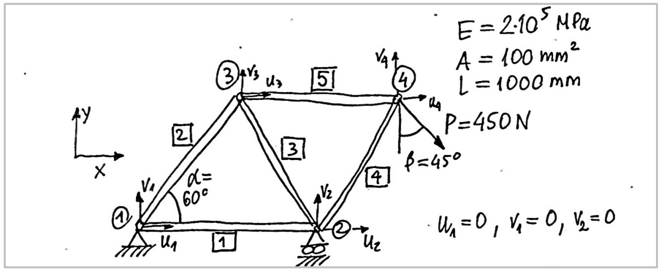
5

$c_5 = 1,\ s_5 = 0,\ \lfloor q_g \rfloor_5 = \lfloor u_3, v_3, u_4, v_4 \rfloor$

$$\underset{4\times4}{[k_g]_5} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_5^*} = \frac{EA}{4l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$





$$\underset{8\times8}{[k_g]_1^*} = \frac{EA}{4L}\begin{bmatrix} 4 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_2^*} = \frac{EA}{4L}\begin{bmatrix} 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_3^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\sqrt{3} & -1 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & -1 & \sqrt{3} & 1 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{3} & -3 & -\sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

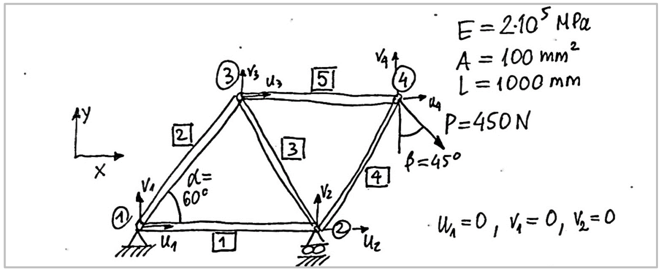
$$\underset{8\times8}{[k_g]_4^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_5^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times8}{[K]} = \sum_{e=1}^{5}[k_g]_e^* = \frac{EA}{4L}\begin{bmatrix} 5 & \sqrt{3} & -4 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ -4 & 0 & 6 & 0 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ 0 & 0 & 0 & 6 & \sqrt{3} & -3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & -1 & \sqrt{3} & 6 & 0 & -4 & 0 \\ -\sqrt{3} & -3 & \sqrt{3} & -3 & 0 & 6 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & -4 & 0 & 5 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times8}{[K]}\cdot\underset{8\times1}{\{q\}} = \underset{8\times1}{\{F\}}$$

+ boundary conditions : $u_1 = 0,\ v_1 = 0,\ v_2 = 0$



$$[K]_{5\times5}\cdot\{q\}_{5\times1}=\{F\}_{5\times1}$$

$$\frac{EA}{4l}\begin{bmatrix} 6 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ -1 & 6 & 0 & -4 & 0 \\ \sqrt{3} & 0 & 6 & 0 & 0 \\ -1 & -4 & 0 & 5 & \sqrt{3} \\ -\sqrt{3} & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}\cdot\begin{Bmatrix} u_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2}P \\ -\frac{\sqrt{2}}{2}P \end{Bmatrix}$$

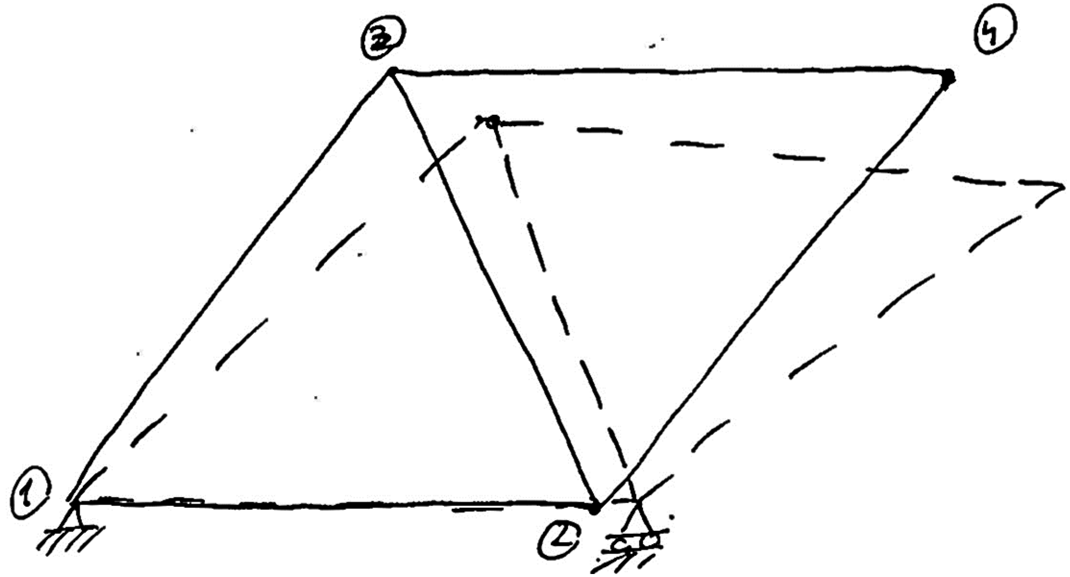
$u_2 = 0.3362\cdot10^{-2}\ mm$

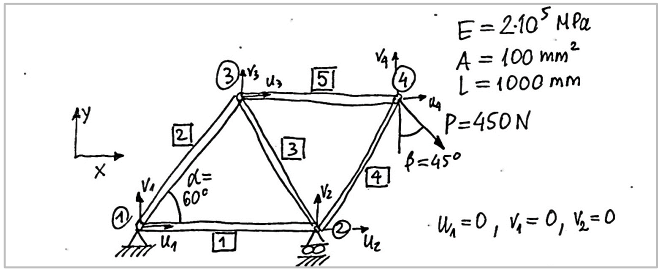
$u_3 = 5.1872\cdot10^{-2}\ mm$

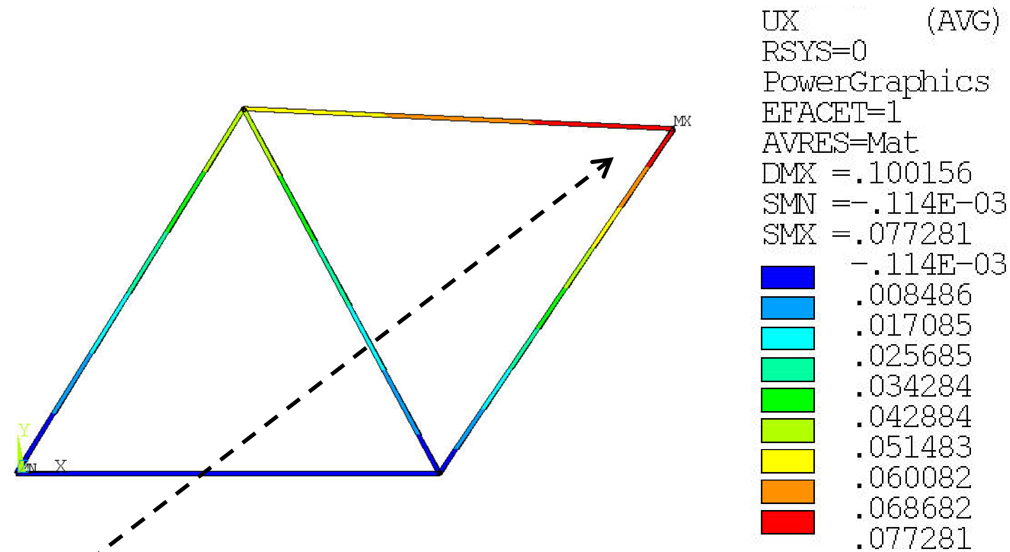
$v_3 = -0.09706\cdot10^{-2}\ mm$

$u_4 = 7.6968\cdot10^{-2}\ mm$

$v_4 = -6.3709\cdot10^{-2}\ mm$







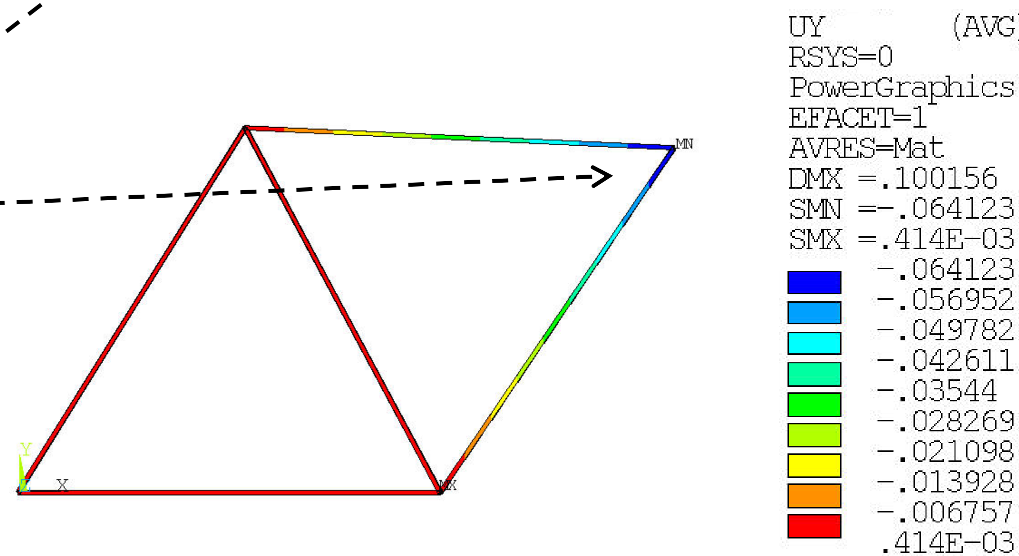
$$u_2 = 0.3362 \cdot 10^{-2}\ mm$$

$$u_3 = 5.1872 \cdot 10^{-2}\ mm$$

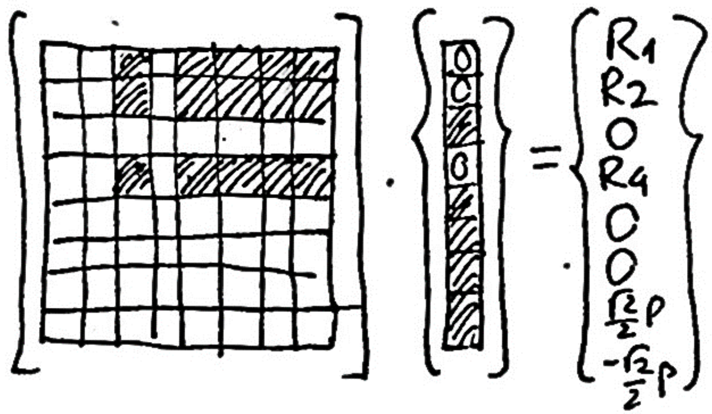
$$v_3 = -0.09706 \cdot 10^{-2}\ mm$$

$$u_4 = 7.6968 \cdot 10^{-2}\ mm$$

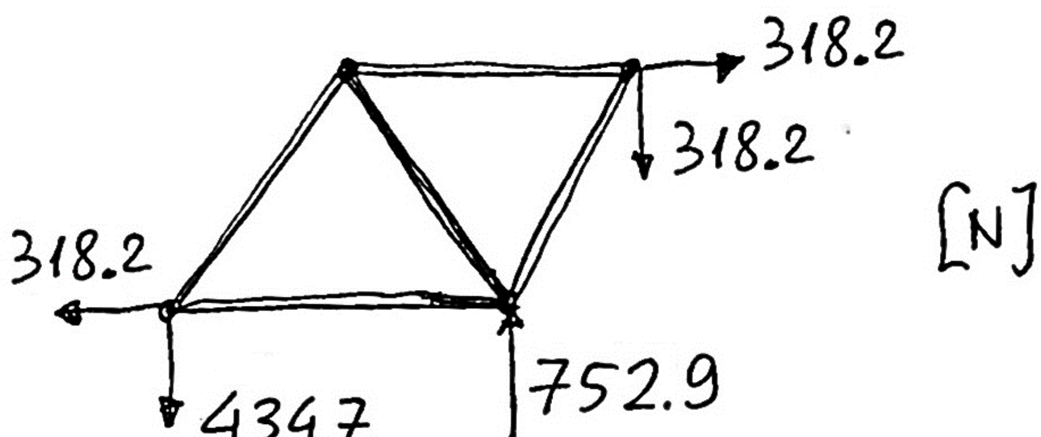
$$v_4 = -6.3709 \cdot 10^{-2}\ mm$$



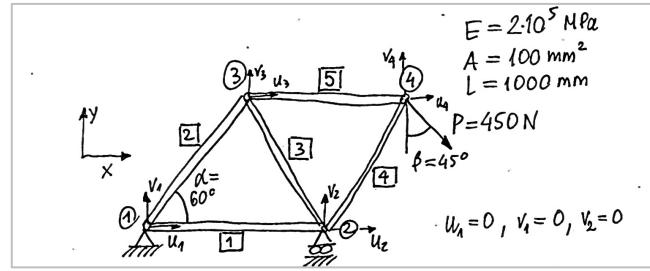
Reakcje



$$\Rightarrow \quad R_1 = -318.2\ N, \quad R_2 = -434.7\ N, \quad R_4 = 752.9\ N$$



Naprężenia i siły wewnętrzne



$$\boxed{1}\quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_{1\,(2\times 1)} = \left[T_t\right]_{1\,(2\times 4)} \cdot \begin{Bmatrix} q_g \end{Bmatrix}_{1\,(4\times 1)} = \underset{(2\times 4)}{\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & c_1 & s_1 \end{bmatrix}} \cdot \underset{(4\times 1)}{\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}_1} = \begin{Bmatrix} 0 \\ 0.33622\cdot 10^{-2} \end{Bmatrix}$$

$$\sigma_1 = \frac{E}{l}\left(q_2 - q_1\right)_1 = 0.67\ \text{MPa}, \quad N_1 = \sigma_1 \cdot A = 67.24\ \text{N}$$

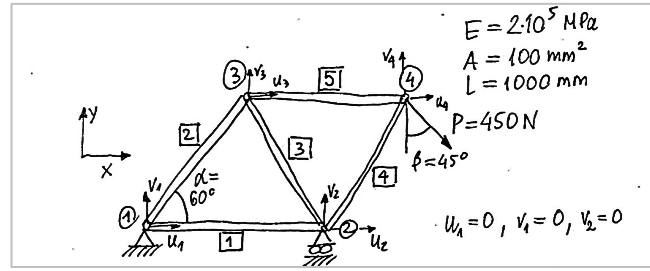
$$\boxed{2}\quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_{2\,(2\times 1)} = \left[T_t\right]_{2\,(2\times 4)} \cdot \begin{Bmatrix} q_g \end{Bmatrix}_{2\,(4\times 1)} = \begin{bmatrix} c_2 & s_2 & 0 & 0 \\ 0 & 0 & c_2 & s_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}_2 = \begin{Bmatrix} 0 \\ 2.5096\cdot 10^{-2} \end{Bmatrix}$$

$$\sigma_2 = \frac{E}{l}\left(q_2 - q_1\right)_2 = 5.02\ \text{MPa}, \quad N_2 = \sigma_2 \cdot A = 502\ \text{N}$$

$$\boxed{3}\quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_{3\,(2\times 1)} = \left[T_t\right]_{3\,(2\times 4)} \cdot \begin{Bmatrix} q_g \end{Bmatrix}_{3\,(4\times 1)} = \begin{bmatrix} c_3 & s_3 & 0 & 0 \\ 0 & 0 & c_3 & s_3 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}_3 = \begin{Bmatrix} -0.16811\cdot 10^{-2} \\ -2.67766\cdot 10^{-2} \end{Bmatrix}$$

$$\sigma_3 = \frac{E}{l}\left(q_2 - q_1\right)_3 = -5.02\ \text{MPa}, \quad N_3 = \sigma_3 \cdot A = -502\ \text{N}$$

(possible buckling)



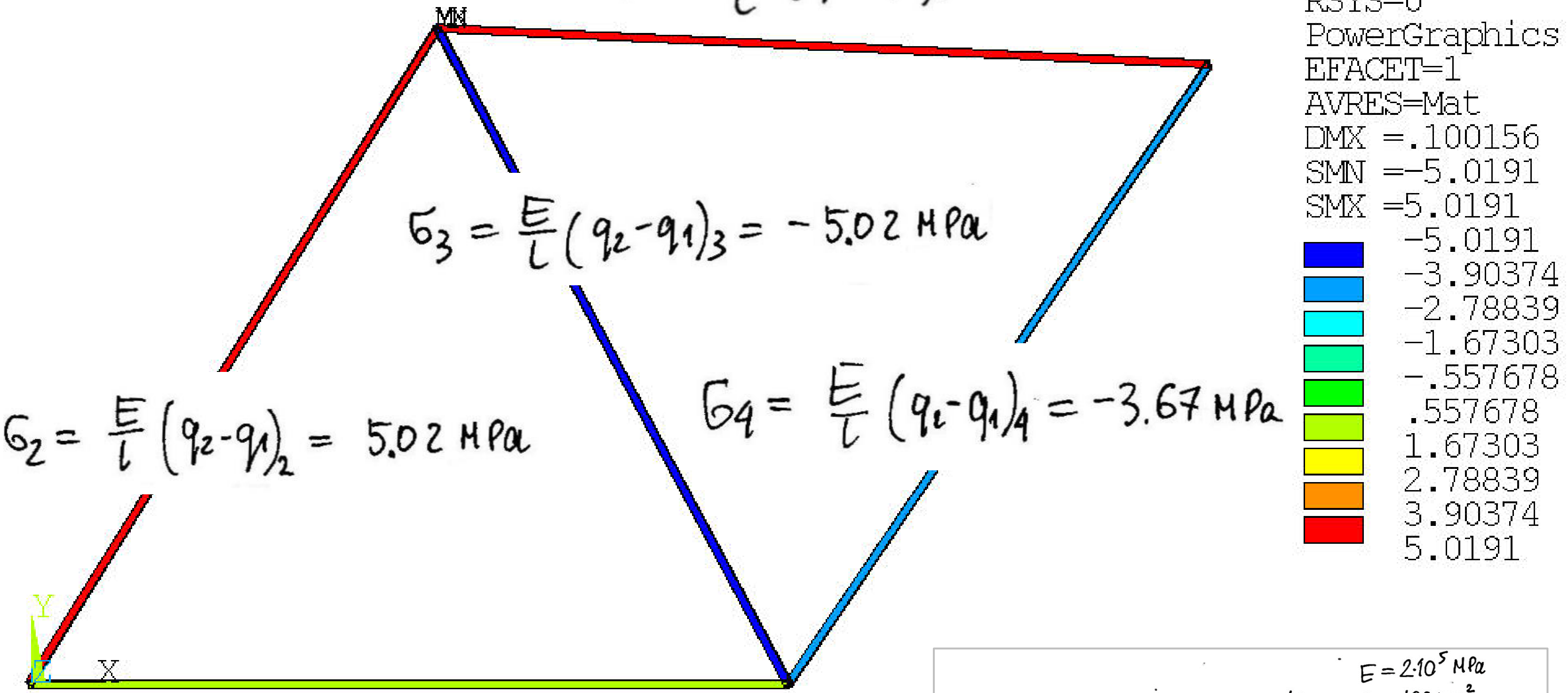
[4] $\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_{4\,(2\times1)} = [T_t]_4 \cdot \{q_g\}_4 = \begin{bmatrix} c_4 & s_4 & 0 & 0 \\ 0 & 0 & c_4 & s_4 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 0.16811 \cdot 10^{-2} \\ -1.66901 \cdot 10^{-2} \end{Bmatrix}$

$\sigma_4 = \frac{E}{L}(q_2 - q_1)_4 = -3.67 \text{ MPa}$, $N_4 = \sigma_4 \cdot A = -367 \text{ N}$ (possible buckling)

[5] $\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_{5\,(2\times1)} = [T_t]_{5\,(2\times4)} \cdot \{q_g\}_{5\,(4\times1)} = \begin{bmatrix} c_5 & s_5 & 0 & 0 \\ 0 & 0 & c_5 & s_5 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 5.18721 \cdot 10^{-2} \\ 7.6968 \cdot 10^{-2} \end{Bmatrix}$

$\sigma_5 = \frac{E}{L}(q_2 - q_1)_5 = 5.02 \text{ MPa}$, $N_5 = 502 \text{ N}$

Naprężenia [MPa]



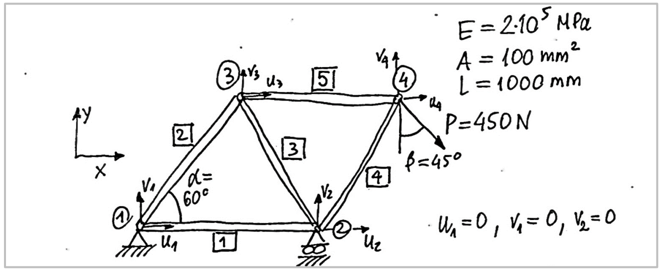
$$\sigma_5 = \frac{E}{l}(q_2 - q_1)_5 = 5.02\ MPa$$

$$\sigma_3 = \frac{E}{l}(q_2 - q_1)_3 = -5.02\ MPa$$

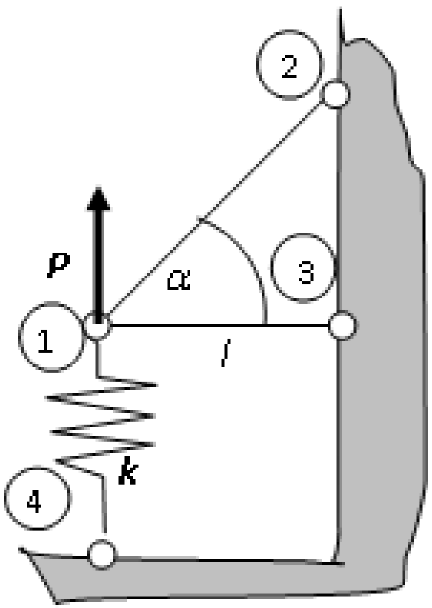
$$\sigma_2 = \frac{E}{l}(q_2 - q_1)_2 = 5.02\ MPa$$

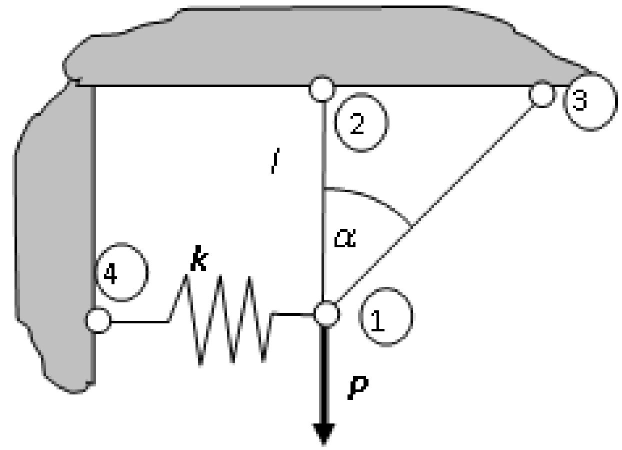
$$\sigma_4 = \frac{E}{l}(q_2 - q_1)_4 = -3.67\ MPa$$

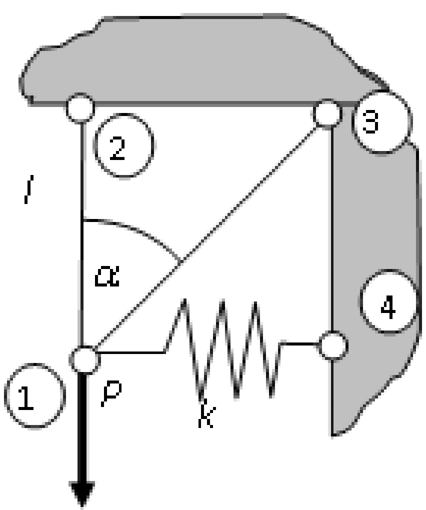
$$\sigma_1 = \frac{E}{l}(q_2 - q_1)_1 = 0.67\ MPa$$



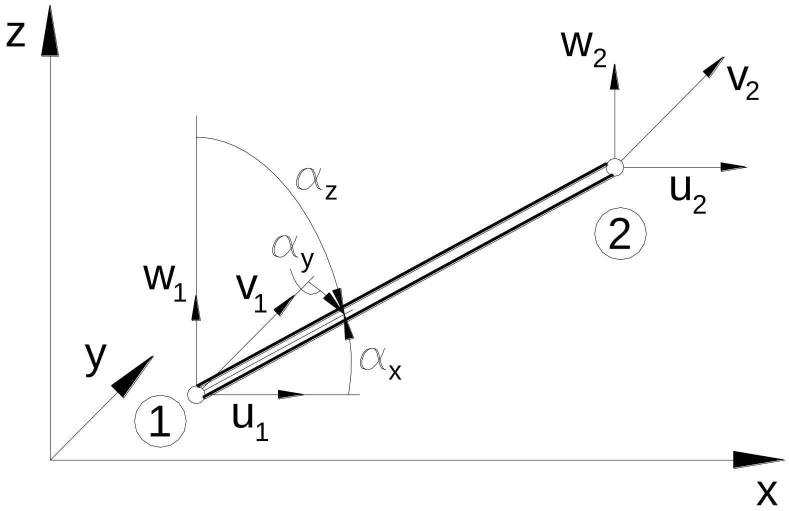
Przykłady zadań







Element skończony pręta kratownicy 3D



$$\{q\}_e = \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix}$$ Globalny wektor parametrów węzłowych

Globalna macierz sztywności pręta kratownicy:

$$\left[k^g\right]_e = \frac{EA}{l_e} \begin{bmatrix} c_x^2 & c_x c_y & c_x c_z & -c_x^2 & -c_x c_y & -c_x c_z \\ c_x c_y & c_y^2 & c_y c_z & -c_x c_y & -c_y^2 & -c_y c_z \\ c_x c_z & c_y c_z & c_z^2 & -c_x c_z & -c_y c_z & -c_z^2 \\ -c_x^2 & -c_x c_y & -c_x c_z & c_x^2 & c_x c_y & c_x c_z \\ -c_x c_y & -c_y^2 & -c_y c_z & c_x c_y & c_y^2 & c_y c_z \\ -c_x c_z & -c_y c_z & -c_z^2 & c_x c_z & c_y c_z & c_z^2 \end{bmatrix}$$

$$c_x = \cos\alpha_x \qquad c_y = \cos\alpha_y \qquad c_z = \cos\alpha_z$$